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# SPHERICALLY SYMMETRIC STATIC BULK VISCOUS FLUID IN ROSEN'S MODIFIED BIMETRIC THEORY OF RELATIVITY

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**ABSTRACT**

In this paper we have studied the static spherically symmetric metric

$ds^2 = g^2(r)dt^2 - dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\phi^2)$ with energy momentum tensor $T_i^j$ for the bulk viscous fluid distribution and we have adopted the procedure of Khadekar and Kondawar (2006) for bulk viscous fluid and obtained the results on the line of Khadekar and Kondawar (2006) which represents the Schwarzschild interior solution.

***KEYWORDS:***

Spherically symmetric; Bulk viscous fluid; Bimetric theory of Relativity; Schwarz child interior solution.

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## 1. INTRODUCTION:

In the development of many areas of physics and astrophysics, exact solutions of Einstein's field equations play a vital role. Schwarzschild (1969) [2] has first provided solution of Einstein field equations of general Relativity, when he published details of the static spherically symmetric vacuum metric that now bears his name. Hereafter many researchers have tried to obtain exact solution of Einstein field equations. But the solutions are not free from singularity, so to remove such unsatisfactory features Rosen (1940)[3] has developed a bimetric theory of gravitation by considering two metric and where background metric $\gamma_{ij}$ represents the geometry and do not interact with the matter. But with this background metric being a flat metric, Rosen could not obtain satisfactorily results, he was getting only vacuum models.So he modified his theory in 1980 [4] replacing flat metric with metric of constant curvature. Hojman and Santamarina (1984)[5], Barger et al. (1907) [6], Khadekar and Kandalkar (2004)[7], have developed a procedure in general relativity to obtain exact analytical solutions of field equations in general relativity. Khadekar and Kondawar (2006) [1] have presented a procedure to obtain general exact analytical solutions of the field equations of Rosen's bimetric relativity for a static spherically symmetric perfect fluid. The general analytic solutions obtained depend upon arbitrary function of the radial coordinate. Here it is shown that interior Schwarzschild's solution is regained in bimetric general relativity. In this paper we have adopted the procedure of Khadekar and Kondawar (2006) for bulk viscous fluid and obtained the results on the line of Khadekar and Kondawar (2006).

## 2. FIELD EQUATIONS , GENERATING FUNCTIONS AND THEIR SOLUTIONS :

The Rosen's bimetric theory of relativity is a modification of Einstein Relativity theory in order to eliminate the singularities appearing in it such as the singularity at the Centre of the black hole or big-bang in cosmology. In bimetric theory of relativity there is a physical metric $g_{ij}$ as in conventional general relativity and there is also a background metric $\gamma_{ij}$ having a curvature tensor $P_{hijk}$ given by

$$P_{hijk} = \frac{1}{a^2}\left(\gamma_{ij}\,\gamma_{\mu\nu} - \gamma_{i\sigma}\,\gamma_{\mu\sigma}\right) \tag{2.1}$$

where a is a constant taken to be the order of the size of the universe, $a \approx 10^{28}$ cm. the field equations of bimetric relativity are taken to be the same as in general relativity. Except for the fact that ordinary derivatives of $g_{ij}$ are replaced by covariant derivatives with respect to $\gamma_{ij}$. Rosen (1980) found that field equations in bimetric relativity can be written in the form of Einstein's field equations but with the additional term on the right hand side

$$G_i^j = S_i^j + T_i^j$$

(2.2)

where $G_i^j = R_i^j - \frac{1}{2} R g_i^j$ is the Einstein tensor ($R_i^j$ is Ricci tensor, $R$ is Ricci scalar curvature), $T_i^j$ is energy stress tensor and $S_i^j = \frac{3}{a^2}(\gamma_{i\alpha} g^{\alpha j} - \frac{1}{2} g_i^j \gamma_{\alpha\beta} g^{\alpha\beta})$ , where a is very large, this term is usually neglected, so that for phenomena in solar system bimetric theory gives agreement with general relativity.However if one has a solution where according to general relativity, $g_{ij}$ has a singularity, then this term cannot be neglected and the field equations of bimetric relativity may give result different from those of general relativity.

We consider the static spherically symmetric system as

$$ds^2 = g^2(r)dt^2 - dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.3}$$

The energy momentum tensor $T_i^j$ for the bulk viscous fluid distribution can be expressed as $T_i^j = (\rho + p)v_i v^j - p g_i^j$ , where $\overline{p} = p - \xi v^j_{;i}$ together with $g_{ij} v_i v^j = 1$ i.e. $v_4 v^4 = 1$, where $v_i$ is the four velocity of the bulk viscous fluid. $\rho, p, \overline{p}, \xi$ are the energy density, isotropic pressure, effective pressure and bulk viscous coefficient respectively.

Therefore

$$T_1^1 = T_2^2 = T_3^3 = -\overline{p} \quad \text{and} \quad T_4^4 = \rho \tag{2.4}$$

For the background metric $\gamma_{ij}$ , we consider De-Sitter Universe of constant curvature as

$$d\sigma^2 = (1 - \frac{r^2}{a^2})dt^2 - (1 - \frac{r^2}{a^2})^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

The non vanishingChristoffel symbols are:

$$\Gamma^1_{22} = -RR', \Gamma^1_{33} = -RR'\sin^2\theta, \Gamma^1_{44} = gg', \Gamma^2_{12} = \frac{R'}{R}, \Gamma^2_{33} = -\sin\theta\cos\theta,$$

$$\Gamma^3_{13} = \frac{R'}{R}, \Gamma^3_{23} = \cot\theta, \Gamma^4_{14} = \frac{g'}{g}$$

Now $R_{ij} = \frac{\partial}{\partial x^i}\Gamma^k_{ik} - \frac{\partial}{\partial x^k}\Gamma^k_{ij} + \Gamma^p_{ip}\Gamma^k_{pj} - \Gamma^p_{ij}\Gamma^k_{pk}$

Therefore, $R_{11} = \frac{\partial}{\partial x^1}\Gamma^k_{1k} - \frac{\partial}{\partial x^k}\Gamma^k_{11} + \Gamma^p_{1p}\Gamma^k_{p1} - \Gamma^p_{11}\Gamma^k_{pk}$

Now applying summation convention on $p$ and $q$ and using nonvanishing christoffel symbols, we obtain

$$R_{11}=2\frac{R''}{R}+\frac{g''}{g}\quad R_{22}=RR''+R'^2+\frac{RR'g'}{g}-1$$

$$R_{33}=(RR''+R'^2+\frac{RR'g'}{g}-1)\sin^2\theta \quad \text{and} \quad R_{44}=-gg''-\frac{2gg'R'}{R}$$

Where ', 'denotes derivative with respect to r.

Then mixed Ricci Tensor $R_i^j\,(i.e. R_i^j=R_{ij}g^{ij})$ is obtained as

$$R_1^1=-\frac{2R''}{R}-\frac{g''}{g},R_2^2=-\frac{R''}{R}-\frac{R'^2}{R^2}-\frac{R'g'}{Rg}+\frac{1}{R^2}=R_3^3,R_4^4=-\frac{g''}{g}-\frac{2R'g'}{Rg}$$

Therefore

$$R=R_1^1+R_2^2+R_3^3+R_4^4$$

$$\therefore R=-4\frac{R''}{R}-2\frac{g''}{g}-2\frac{R'^2}{R^2}-4\frac{R'g'}{Rg}+\frac{2}{R^2}$$

(2.5)

Then $G_i^j=R_i^j-\frac{1}{2}Rg_i^j$ gives

$$G_1^1=\frac{R'^2}{R^2}+2\frac{R'g'}{Rg}-\frac{1}{R^2},G_2^2=\frac{R''}{R}+\frac{g''}{g}+\frac{R'g'}{Rg}=G_3^3,G_4^4=2\frac{R''}{R}+\frac{R'^2}{R^2}-\frac{1}{R^2} \tag{2.6}$$

Now

$$S_\mu^\nu=\frac{3}{a^2}[\gamma_{\mu\alpha}g^{\alpha\nu}-\tfrac{1}{2}\delta_\mu^\nu\gamma_{\alpha\beta}g^{\alpha\beta}]$$

$$\therefore S_1^1=\frac{3}{a^2}\{(\gamma_{11}g^{11})-\tfrac{1}{2}\delta_1^1(\gamma_{11}g^{11}+\gamma_{22}g^{22}+\gamma_{33}g^{33}+\gamma_{44}g^{44})$$

$$=\frac{3}{a^2}\{\tfrac{1}{2}(1-\tfrac{r^2}{a^2})^{-1}-\tfrac{r^2}{g^2}-\tfrac{1}{2}(1-\tfrac{r^2}{a^2})\tfrac{1}{g^2}\}$$

$$=\frac{3}{2(a^2-r^2)}-\frac{3r^2}{a^2g^2}-\frac{3}{2a^2g^2}(1-\frac{r^2}{a^2})$$

$$=-\frac{3}{2a^2g^2}$$ , neglecting other terms.

Similarly,

$S_2^2 = -\frac{3}{2a^2g^2} = S_3^3 \quad ,S_4^4 = \frac{3}{2a^2g^2}$. Other $S_j^i = 0$

(2.7)

$\therefore G_\mu^\nu = S_\mu^\nu + T_\mu^\nu$

$\Rightarrow G_1^1 = S_1^1 + T_1^1, G_2^2 = S_2^2 + T_2^2, \qquad G_3^3 = S_3^3 + T_3^3 \qquad , \qquad G_4^4 = S_4^4 + T_4^4$

(2.8)

Using equations (2.4),(2.6) and (2.7) in (2.8), we obtain

$$\frac{R'^2}{R^2} + 2\frac{R'g'}{Rg} - \frac{1}{R^2} = \bar{p} - \frac{3}{2a^2g^2},$$

(2.9)

$$\frac{R''}{R} + \frac{g''}{g} + \frac{R'g'}{Rg} = \bar{p} - \frac{3}{2a^2g^2},$$

(2.10)

$$2\frac{R''}{R} + \frac{R'^2}{R^2} - \frac{1}{R^2} = \frac{3}{2a^2g^2} - \rho$$

(2.11)

The consequences of energy momentum conservation, $T^\mu_{\nu;\mu} = 0$ leads to

$$\bar{p}' + (\rho + \bar{p})\frac{g'}{g} = 0$$

It should be noted that the background metric does not in these field equation.

One can define the effective density and pressure (Hurpaz and Rosen 1985) as

$$\rho_E = \rho - \frac{3}{2a^2g^2}, \qquad \bar{p}_E = \bar{p} - \frac{3}{2a^2g^2}$$

So that the equation is given by

$$2\frac{R''}{R} + \frac{R'^2}{R^2} - \frac{1}{R^2} = -\rho_E \qquad ,$$

(2.12)

$$\frac{R'^2}{R^2} + 2\frac{R'g'}{Rg} - \frac{1}{R^2} = \bar{p}_E \qquad ,$$

(2.13)

$$\frac{R^{''}}{R}+\frac{g^{''}}{g}+\frac{R^{'}g^{'}}{Rg}=\overline{p}_E$$

(2.14)

These equations looks like Einstein equation with $\rho$ and $p$ replaced by $\rho_E$ and $\overline{p}_E$ respectively, we get

$$\overline{p_E}^{'}+(\rho_E+\overline{p_E})\frac{g^{'}}{g}=0$$

(2.15)

From equation (3.2.12) , we obtain

$$R^{'2}=1-\frac{2m_E(R)}{R}$$

(2.16)

$$\Rightarrow R^{'}=\frac{dR}{dr}=\sqrt{1-\frac{2m_E(R)}{R}}$$

$$\therefore \frac{dr}{dR}=\frac{1}{\sqrt{1-\frac{2m_E(R)}{R}}}$$

$$\Rightarrow dr=\frac{dR}{\sqrt{1-\frac{2m_E(R)}{R}}}$$

$$\Rightarrow r=\int\frac{dR}{\sqrt{1-\frac{2m_E(R)}{R}}}$$ , On integration

Using equations (2.15) and (2.16) in equation (2.13) ,we obtain

$$(R-2m_E)[\frac{1}{R^3}-\frac{2}{R^2(\rho_E+p_E)}\frac{dp_E}{dR}]=p_E+\frac{1}{R^2}$$

(2.17)

We define a function $G(R)$ as

$$G(R)=-\frac{(R-2m_E)}{p_E+\frac{1}{R^2}}$$

(2.18)

Also,

$$\frac{g'}{g} = -\frac{p_E'}{(\rho_E + p_E)}$$

$$\Rightarrow g = \exp[-\int \frac{dp_E/dR}{(\rho_E + p_E)} dR + g_0] \quad ,$$

On integrationand taking $g_0$ is constant of integration.

$$\Rightarrow g = A_0 \exp[-\int \frac{dp_E/dR}{(\rho_E + p_E)} dR]$$

$$\Rightarrow g^2 = A_0^2 \exp[-2\int \frac{dp_E/dR}{(\rho_E + p_E)} dR]$$

Using equation (2.17) and the value of $G(R)$ by (2.18), we obtain

$$g^2 = \frac{A_0^2}{R} \exp[-\int \frac{R^2}{G} dR]$$

Hence, equation (2.3) becomes

$$ds^2 = \frac{A_0^2}{R} \exp[-\int \frac{R^2}{G} dR] dt^2 - \frac{dR^2}{1 - \frac{2m_E(R)}{R}} - R^2(r)(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.19}$$

which represents the Schwarzschild interior solution.

**3. CONCLUSION:**

Here we have studied the static spherically symmetric metric withenergy momentum tensor for the bulk viscous fluid distribution and we have obtained the solution which represents the Schwarzschild interior solution.

**REFERENCES:**

[1] Khadekar and Kondawar (2006)
[2] Schwarzschild (1969)
[3] Rosen (1940)
[4] Rosen (1980)
[5] Hojman and Santamarina (1984),
[6] Barger et al. (1907),
[7] Khadekar and Kandalkar (2004)
[8] Hurpaz and Rosen (1985)